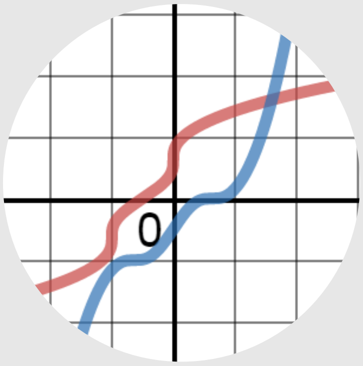

Inverse Function

The Function Reverser

Notation: $f^{-1}(x)$

Swaps inputs and outputs

Only exists when $f(x)$ passes the horizontal line test

Compose with $f(x)$ to get x.

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About me

The inverse function $f^{-1}(x)$ of a function $f(x)$ is a function such that $f(x) = y$ if and only if $x = f^{-1}(x)$. The input of $f(x)$ is the output of $f^{-1}(x)$ while the output of $f(x)$ is the input of $f^{-1}(x)$. A function f has an inverse only when f is a one-to-one function. Visually, this means that f has an inverse only if the graph of f passes the horizontal line test. We can think of f and f^{-1} as "undoing" each other. $f\left(f^{-1}(x)\right) = x$ and $f^{-1}\left(f(x)\right) = x$.

Attributes

Usefulness in Solving Problems

Usefulness in Interpreting Problems

Difficulty

Prevalence

Used in Calculus

Capabilities

- Able to reverse the roles of the input and output of a function.
- Undo the affects of a function by composing with that function.
- Solve tor a variable x in the function

$$y = f(x)$$

  by composing on the left by f^{-1}.
- Provide a formula for the function obtained by reflecting across the line $y = x$ in the plane.

Common Mistakes

- The number -1 in the superscript is not an exponent. It does not mean reciprocal. Instead it refers to the fact that f^{-1} is the inverse function of f under function composition.
- Not every function is named f. The inverse of $g(x)$ is denoted $g^{-1}(x)$. Be sure to use the appropriate function name.
- Not every function has an inverse. For example, the function $h(x) = x^2$ only has an inverse if you restrict the domain of $h(x)$ to be positive. In that case, $h^{-1}(x) = \sqrt{x}$.

Applications

Temperature Conversion The function $f(x) = \frac{9}{5}x + 32$ takes an input temperature of x degrees Celcius and outputs the temperature $f(x)$ in degrees Fahrenheit. The inverse function $f^{-1}(x) = \frac{5}{9}(x-32)$ takes an input of x degrees Fahrenheit and outputs the temperature in $f^{-1}(x)$ degrees Celcius.

Dating Ages The minimum dating age formula $A(x) = \frac{1}{2}x + 7$ inputs someone's age x and outputs the minimum age of an individual they could date (says some rule on the internet). The function $A^{-1}(x) = 2(x-7)$ inputs the age of an individual and outputs the maximum age of someone they could date.

Geometry A cube of side length x has a surface area given by $A(x) = 6x^2$ and a volume given by $V(x) = x^3$. From these formulas we can derive many facts.

- The formula $A^{-1}(x) = \sqrt{\frac{x}{6}}$ inputs the surface area of the cube and outputs the side length.
- The formula $V^{-1}(x) = \sqrt[3]{x}$ inputs the volume of the cube and outputs the side length.
- The function that inputs the cube's volume and ouputs the surface area is $A\left(V^{-1}(x)\right) = 6\left(\sqrt[3]{x}\right)^2$.
- The function that inputs the cube's surface area and ouputs the volume is $V\left(A^{-1}(x)\right) = \left(\sqrt{\frac{x}{6}}\right)^3$.

Methods

How do we find an inverse function $f^{-1}(x)$ from a function $f(x)$?

Steps

1. Start with an expression $y = f(x)$.
2. Swap variables to get $x = f(y)$.
3. Solve for y.
4. Replace y with $f^{-1}(x)$.

Example

1. $y = (2x-3)^3$.
2. $x = (2y-3)^3$.
3. $y = \frac{\sqrt[3]{x}+3}{2}$.
4. $f^{-1}(x) = \frac{\sqrt[3]{x}+3}{2}$.