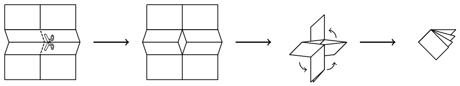
# Chapter 2 Study Guide

Name: ____________________



## 2.1 Power Rule

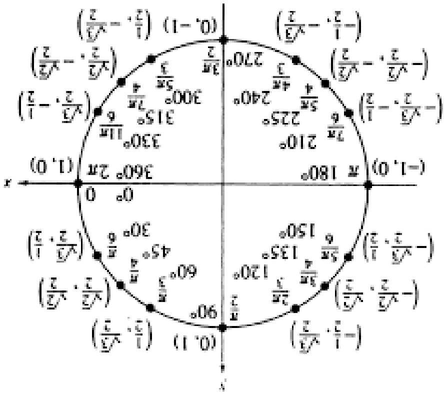
- $\frac{d}{dx}[x^n] = nx^{n-1}$, $n \neq 0$.
- $\frac{d}{dx}[a^x] = a^x \ln(a)$.
- $\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$.
- $\frac{d}{dx}[cf(x)] = cf'(x)$.

**Example:** Find the derivative of

$$f(x) = 2x^5 - \frac{3}{x} + \sqrt{x} + 4^x + 17.$$

Solution: $f'(x) = 10x^4 + 3x^{-2} + \frac{1}{2}x^{-1/2} + 4^x \ln(4)$.

## 2.2 Sine and Cosine Derivatives




- $\frac{d}{dx}[\sin(x)] = \cos(x)$.
- $\frac{d}{dx}[\cos(x)] = -\sin(x)$.

**Example:** Find the derivative of

$$f(x) = x^2 + 5\cos(x) - 4\sin(x).$$

Solution: $f'(x) = 2x - 5\sin(x) - 4\cos(x)$.



## 2.3 Product and Quotient Rules

- $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$.
- $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$.

  The derivative of a product is the derivative of the first times the second plus the first times the derivative of the second.

  The derivative of a quotient is the derivative of the top times the bottom minus the top times the derivative of the bottom, all over the bottom squared.

**Example:** Find the derivative of

$$f(x) = x^3 \sin(x).$$

Solution: $f'(x) = 3x^2 \sin(x) + x^3 \cos(x)$.
**Example:** Find the derivative of

$$g(x) = \frac{2^x}{x^2+7}.$$

Solution: $g'(x) = \frac{2^x \ln(2)(x^2+7) - 2^x(2x)}{(x^2+7)^2}$.



## 2.4 Other Trig Derivatives

- $\frac{d}{dx}[\tan(x)] = \sec^2(x)$.
- $\frac{d}{dx}[\cot(x)] = -\csc^2(x)$.
- $\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$.
- $\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$.

**Example:** Find the derivative of

$$f(x) = x^2 \sec(x).$$

Solution: $f'(x) = 2x\sec(x) + x^2\sec(x)\tan(x)$.
**Example:** Find the derivative of

$$g(x) = 5\tan(x) + 4 - 3\cot(x).$$

Solution: $g'(x) = 5\sec^2(x) + 3\csc^2(x)$.
**Example:** Find the derivative of

$$h(x) = e^x + 4\csc(x).$$

Solution: $h'(x) = e^x - 4\csc(x)\cot(x)$.



## 2.5 Chain Rule

- $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$.

  The deriviative of a composition is the derivative of the outer evaluated at the inner times the derivative of the inner.

**Example:** Find the derivative of

$$f(x) = \sin(e^x).$$

Solution: $f'(x) = \cos(e^x)e^x$.
**Example:** Find the derivative of

$$g(x) = xe^{4x}.$$

Solution: $g'(x) = e^{4x} + xe^{4x} \cdot 4$.
**Example:** Find the derivative of

$$h(x) = \sec(5^x).$$

Solution: $h'(x) = \sec(5^x)\tan(5^x)5^x\ln(5)$.



## 2.6 Inverse Function Derivatives

- $\frac{d}{dx}[\arctan(x)] = \frac{1}{1+x^2}$.
- $\frac{d}{dx}[\arcsin(x)] = \frac{1}{\sqrt{1-x^2}}$.
- $\frac{d}{dx}[\log_a(x)] = \frac{1}{x\ln(a)}$.
- $\left(f^{-1}\right)'(a) = \frac{1}{f'(f^{-1}(a))}$.

**Example:** Find the derivative of

$$f(x) = \ln(\ln(x)).$$

Solution: $f'(x) = \frac{1}{\ln(x)} \cdot \frac{1}{x}$.
**Example:** Find the derivative of

$$g(x) = x\arctan(x).$$

Solution: $g'(x) = \arctan(x) + x \cdot \frac{1}{1+x^2}$.
**Example:** Find the slope of $f^{-1}(x)$ at (2,1) for the function $f(x) = \ln(x) + 2x$.
Solution: The slope of $f^{-1}$ at (2,1) is the reciprocal of the slope of $f$ at (1,2).

$$\left(f^{-1}\right)'(2) = \frac{1}{f'(1)} = \frac{1}{3}$$



## 2.7 Implicit differentiation

Differentiate like normal. Solve for $\frac{dy}{dx}$. Remember to use the chain rule when it applies.
**Example:** Find the slope of the curve $xy^2 + e^y + x = 3$ at (2,0).
Solution: Remember the product rule.
$1 \cdot y^2 + x(2y)\frac{dy}{dx} + e^y \frac{dy}{dx} + 1 = 0$
$\implies (2xy + e^y)\frac{dy}{dx} = -1 - y^2$
$\implies \frac{dy}{dx} = \frac{-1-y^2}{2xy+e^y}$. At (3,0) the slope is $m = \frac{-1-0^2}{2(2)(0)+e^0} = -1$.
**Example:** Find the slope of the curve $(x+2y)^2 = 25x$ at (1,2).
Solution: Carefully use the chain rule.
$2(x+2y)(1+2\frac{dy}{dx}) = 25$
$\implies 2(x+2y) + 4(x+2y)\frac{dy}{dx} = 25$
$\implies \frac{dy}{dx} = \frac{25-2(x+2y)}{4(x+2y)}$. At (1,2) the slope is $m = \frac{25-2(1+2(2))}{4(1+2(2))} = \frac{3}{4}$.



## 2.8 L'Hopital's Rule

**L'Hopital's Rule:** If a limit $\lim\limits_{x\to c} \frac{f(x)}{g(x)}$ is of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}.$$

**Example:** Evaluate $\lim\limits_{x\to 2} \frac{2^x - 4}{x^2 - 2x}$.
Solution:

$$\lim_{x\to 2} \frac{2^x-4}{x^2-2x} = \lim_{x\to 2} \frac{2^x \ln(2)}{2x-2} = 2\ln(2).$$

**Example:** Evaluate $\lim\limits_{x\to\infty} \frac{e^x}{x^2+4x-1}$.
Solution:

$$\lim_{x\to\infty} \frac{e^x}{x^2+4x-1} = \lim_{x\to\infty} \frac{e^x}{2x+4} = \lim_{x\to\infty} \frac{e^x}{2} = \infty.$$